SUPPLEMENTAL MATERIAL B

Generalization for "*n*" (*n*>2) limiting substrates kinetic

To generalize the procedure to estimate the net rate of biofilm growth for a system with *n* substrates interacting kinetic model, let us assume that i = 1, is the key component (more limiting species). Taking into account the procedure developed for Double-Monod interacting kinetics, the dimensionless mass balances for each species will be:

For $i = 1$ (key substrate)

$$\frac{d}{dx^*} D^*_{f1} \frac{dC^*_1}{dx^*} = \phi^2 \; X^*_f \; r^* \qquad \text{(B-1)}$$

For $i = 2 \;\; to \;\; n$ substrates

$$\frac{d}{dx^*} D^*_{fi} \frac{dC^*_i}{dx^*} = \phi^2 \; \Gamma_i \; X^*_f \; r^* \qquad \text{(B-2)}$$

where

$$\Gamma_i = \nu_i \frac{C_{1s} \; \overline{D}_{f1}}{C_{is} \; \overline{D}_{fi}} \qquad \text{with} \qquad \nu_i = \frac{Y_1}{Y_i} \qquad \text{(B-3)}$$

The Thiele modulus ϕ, becomes

$$\phi^2 = \frac{L_f^2 \; \overline{X}_f}{\overline{D}_{f1} \; C_{1s} \; Y_1} \; q_{\max} \prod_{i=1}^{n} \frac{C_{is}}{(K_i + C_{is})} = \frac{L_f^2 \; \overline{X}_f}{\overline{D}_{f1} \; C_{1s} \; Y_1} r_s \qquad \text{(B-4)}$$

where

$$r = q_{\max} \prod_{i=1}^{n} \frac{C_i}{(K_i + C_i)} \qquad \text{(B-5)}$$

and

$$r_s = q_{\max} \prod_{i=1}^{n} \frac{C_{is}}{(K_i + C_{is})} \quad \text{(B-6)}$$

The dimensionless boundary conditions for the differential equations (B-1) and (B-2) are

$$\text{At} \quad x^* = 1 \qquad C_i^* = 1 \qquad \text{for } i = 1 \quad to \quad n \quad \text{(B-7)}$$

$$\text{At} \quad x^* = 0 \qquad \frac{dC_i^*}{dx^*} = 0 \qquad \text{for } i = 1 \quad to \quad n \quad \text{(B-8)}$$

Considering that all substrates obey relation (19) and taking into account the boundary conditions (B-7) and (B-8), the relation between the dimensionless concentrations of any species with the key substrate concentration C_1^*, is found to be:

$$C_i^* = \Gamma_i \ (C_1^* - 1) + 1 \quad \text{(B-9)}$$

The necessary condition for substrate $i = 1$ to be the key component is that Γ_i (for $i = 2$ to n) should be less than one.

Therefore, the kinetic expression (r^*) results:

$$r^* = \frac{r}{r_s} = \prod_{i=1}^{n} (\beta_i + 1) \frac{\Gamma_i \ C_1^*}{[\beta_i + \Gamma_i \ (C_1^* - 1) + 1]} \quad \text{(B-10)}$$

Also

$$\left(\frac{dr^*}{dC_1^*}\right)_{C_1^*=1} = r^{*'}(1) = \sum_{i=1}^{n}\frac{\Gamma_i\ \beta_i}{(\beta_i+1)} \tag{B-11}$$

Asymptotic solutions for the effectiveness factor

The asymptotic solutions for $\phi << 1$ and $\phi >> 1$, necessary for the application of Equation (33) for multiple (n >2) substrate limitation Monod kinetics, are the following:

For $\phi << 1$, the solution is

$$\eta = 1 - \sigma_m\ \phi^2 \tag{B-12}$$

where the parameter σ_m for multiple-substrate interacting kinetics is given by

$$\sigma_m = \frac{b\,\Psi^2}{\overline{X}_f^2\ \ c\ \ (0.2218)}\left[\sum_{i=1}^{n}\frac{\Gamma_i\ \beta_i}{(\beta_i+1)}\right](F) \tag{B-13}$$

where parameter F is given by Equation (A-10) of the Supplemental Material A.

Under the condition $\phi >> 1$:

$$\frac{D_{f1}^*(1)}{\phi^2}\left(\frac{dC_1^*}{dx^*}\right)_{x^*=1} = \eta \tag{B-14}$$

and

$$\left(\frac{dC_A^*}{dx^*}\right)_{x^*=1}^2 = \phi^2\ \frac{2\ X_f^*(1)}{D_{f1}^*(1)}\left[\int_0^1\left(\prod_{i=1}^{n}(\beta_i+1)\ \frac{\Gamma_i\ C_1^*}{[\beta_i+\Gamma_i(C_1^*-1)+1]}\right)dC_1^*\right] \tag{B-15}$$

Defining:

$$I_m = \int_0^1 \left(\prod_{i=1}^{n} (\beta_i + 1) \frac{\Gamma_i \ C_1^*}{[\beta_i + \Gamma_i (C_1^* - 1) + 1]} \right) dC_1^* \qquad \text{(B-16)}$$

$$\eta = \frac{\left(2 \ D_{f1}^*(1) \ X_f^*(1) \ I_m\right)^{1/2}}{\phi} = \frac{\rho_m}{\phi} \qquad \text{(B-17)}$$